

APRIL/MAY 2025

**23PMA13 — ORDINARY DIFFERENTIAL
EQUATIONS**

Time : Three hours

Maximum : 75 marks



SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

Find all solutions ϕ of $y'' + y = 0$ satisfying $\phi(0) = 1$ and $\phi\left(\frac{\pi}{2}\right) = 2$.

2. Determine whether the function $\phi_1(x) = x$ and $\phi_2(x) = xe^x$, $x \in (-\infty, \infty)$ are linearly dependent or independent.
3. Find all solutions of $y''' - 8y = 0$.
4. State the Existence Theorem of the solution of the IVP involving an n th order homogeneous ordinary differential equation with constant coefficient.

5. Write down the Chebyshev equation.
6. Define Analytic function.
7. Define regular singular point.
8. Write down the n-th Laguerre polynomial.
9. Find all real valued solutions of $y' = x^2 y$.
10. Show that the function $f(x,y) = y^{2/3}$ even though continuous on $R: |x| \leq 1, |y| \leq 1$ does not satisfy Lipschitz condition there.



SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Let ϕ_1, ϕ_2 are any two linearly independent solution of $L(y) = 0$ on an interval I. Show that every solution ϕ of $L(y) = 0$ can be written uniquely as $\phi = c_1 \phi_1 + c_2 \phi_2$ where c_1, c_2 are constant.

Or

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20. Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle $R: |x - x_0| \leq a, |y - y_0| \leq b$.

Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ in } R.$$



- (b) By the method of variation of constants find all the solution of the equation

$$y'' + y = \tan x \left(-\frac{\pi}{2} < x < \frac{\pi}{2} \right).$$

12. (a) Using the annihilator method find a particular solution of $y'' + 9y = x^2 e^{3x}$.

Or

- (b) Let $\phi_1, \dots, \phi_n$ be n linearly independent solution of $L(y) = 0$ on an interval I . If $c_1, \dots, c_n$ are any constants $\phi = c_1 \phi_1 + \dots + c_n \phi_n$ is a solution, and every solution may be represented in this form – Prove.

13. (a) Show that there exists n linearly independent solutions of $L(\phi) = 0$.

Or

- (b) One solution of $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi(x) = x$. Find a basis for the solution for $x > 0$.

14. (a) Show that

$$J_{\alpha-1}(x) - J_{\alpha+1}(x) = 2J_{\alpha}^1(x)$$

$$J_{\alpha-1}(x) + J_{\alpha+1}(x) = \frac{2\alpha}{x} J_{\alpha}(x)$$

Or

- (b) Compute the indicial polynomials and their roots of the equations $x^2 y'' + (x + x^2) y' - y = 0$.

15. (a) Show that the equation $y' = \frac{3x^2 - 2xy}{x^2 - 2y}$ is exact and hence solve it.

Or

- (b) Find the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ of the equation $y' = 1 + xy$, $y(0) = 0$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Consider the equation $y'' + y' - 6y = 0$

- (a) Compute the solution ϕ satisfying $\phi(0) = 1$, $\phi'(0) = 0$.
 (b) Compute the solution ψ satisfying $\psi(0) = 0$, $\psi'(0) = 1$.
 (c) Compute $\phi(1)$ and $\psi(1)$.

17. Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0$ on an interval I containing a point x_0 . Then prove that for all x in I $\|\phi(x_0)\| e^{-K|x-x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\| e^{K|x-x_0|}$ where $K = 1 + |a_1| + \dots + |a_n|$.

18. Find two linearly independent power series (in powers of x) of the equation $y'' - x^2 y = 0$.

19. Find a solution $\phi(x) = x^r \sum_{k=0}^{\infty} c_k x^k$ ($x > 0$) for the equation $2x^2 y'' + (x^2 - x) y' + y = 0$.

